Experimental studies of scaling behavior of a quantum Hall system with a tunable Landau level mixing

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Temperature dependence of the longitudinal and Hall resistances is studied in the regime of localization-delocalization transition. We carry out measurements of scaling exponent κ in the Landau level mixing region at several filling factors. The localization exponent γ is extracted using an approach based on the variable-range hopping theory. The values of γ and κ are found to be universal and independent of the filling factor in our sample. We can conclude that although Landau level mixing can change the degeneracy of a quantum Hall state, the value of the scaling exponent remains the same for a given sample that contains a fixed disorder profile.

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The localization problem has been studied intensively in the past decades.[1,2] It is well accepted that there is no true delocalization state in the noninteraction of two-dimensional electron gas (2DEG) provided with disorder. However when a strong external magnetic field is applied, a delocalized state is formed at the center of the Landau level (LL), while the electronic state is localized away from this discrete energy. The coexistence of localized and delocalized states is essential for the quantum Hall effect. Phase transitions between localized and delocalized states will occur when the Fermi level is swept from one Landau level to another. The relative position of the Fermi level and Landau levels can be experimentally changed by varying the external magnetic field and/or the carrier density. Since this phase transition is considered as a continuous quantum phase transition, finite-size scaling theory[3] can be applied in the critical regime, where the resistance tensor scales as $R_{uv}=R_c f(L/\xi)$ for a sample of finite size L. Here f is a scaling function which can be derived from the microscopic calculation. The localization length ξ diverges as the Fermi level approaches the center of a Landau level E_c as $\xi \propto |E-E_c|^{-\gamma}$ with an exponent γ. The interaction-induced quantum phase coherence length sets the effective sample size L and its temperature dependence of $L \propto T^{-p/2}$. Then one obtains $R_{uv}=R_c f(|B-B_c|T^{-\kappa})$, the scaling function of both the longitudinal resistance R_{xx} and the Hall resistance R_{xy}, where the scaling exponent is expressed as $\kappa = p/2\gamma$. Approaching zero temperature, the maximum slope in the Hall resistance R_{xy} with varying magnetic field B diverges as a power law $\frac{dR_{xy}}{dB}|_{B_c} \propto T^{-\kappa}$, while the half width for the longitudinal resistance R_{xx} vanishes as $\Delta B \propto T^{\kappa}$.[4]

Despite the intense studies of the scaling behaviors of the quantum Hall systems,[4–12] several issues are still unsettled. For example, when the electron spin is unresolved (i.e., the two spin states of the Landau levels are mixed), the interaction of electrons in different Landau levels complicate the problem and draw into question whether this universality of scaling behavior can be preserved. Experimentally, there was an indication that the exponent κ can be changed by a factor of 2 when the system becomes spin degenerate.[13,14] On the other hand, theoretical works,[15,16] based on two different models, concluded that Landau level mixing will not change the universality of this phase transition. The earlier experimental studies on the localization-delocalization phase transition in the Landau level mixing regime is focused on the spin unresolved plateau transitions,[13,14] which involves the adjacent Landau levels with spin degeneracy. The experiment presented in this Brief Report is carried out in a two-subband system in which two LLs with different subband, Landau level index, or spin state can be controllably mixed either by varying the electron density or by changing the magnetic field. The main objective of the study is to determine experimentally the contribution of the Landau level mixing on the scaling behavior.

The sample we investigated is grown by molecular-beam epitaxy and consists of a symmetrical modulation-doped 24-mm-wide single GaAs quantum well bounded on each side by Si δ-doped layers of AlGaAs. Heavy doping creates a very dense 2DEG, resulting in the filling of two subbands in the well when at low temperature. As determined from the Hall resistance data and Shubnikov-de Haas oscillations in the longitudinal resistance, the total density is $n=8.0 \times 10^{11}$ cm^{-2}, where the first and the second subbands have a density of $n_1=6.1\times 10^{11}$ cm^{-2} and $n_2=1.9\times 10^{11}$ cm^{-2}. The sample has a low-temperature mobility $\mu=4.1 \times 10^5$ cm^2/V s, which is extremely high for a 2DEG with two filled subbands. A 100-μm-wide Hall bar with 270 μm between voltage probes was patterned by standard lithography techniques. A NiCr top gate is evaporated on the top of the sample, approximately 350 mm away from the center of the quantum well. By applying a negative gate voltage on the NiCr top gate, the electron density can be tuned continuously. Magnetotransport measurements were carried out in an Oxford Top-Loading Dilution Refrigerator with a base temperature of 15 mK. To measure the longitudinal and Hall resistances, R_{xx} and R_{xy}, we used a standard ac lock-in technique with electric current ranging from 10 to 100 nA at a frequency of 11.3 Hz. We carefully checked for heating effects by varying the amplitude of the current at the same

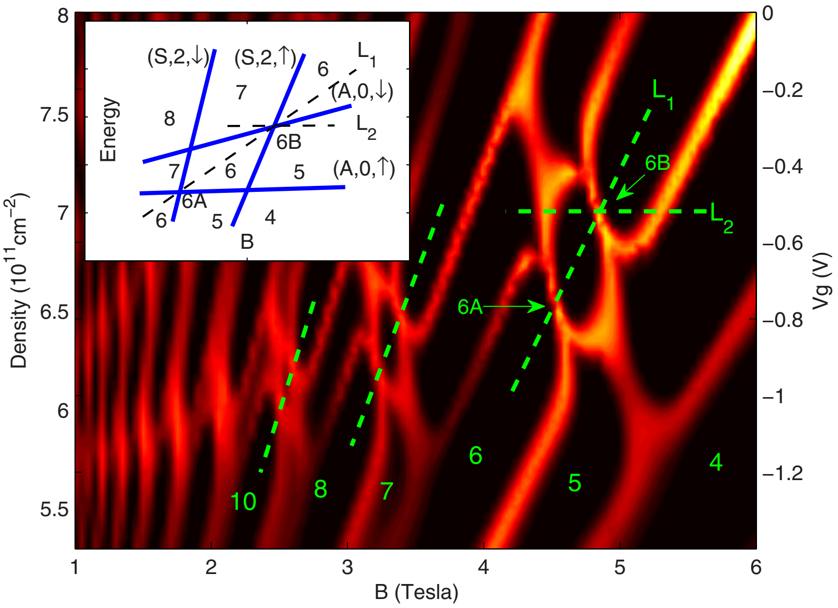

FIG. 1. (Color online) The phase diagram of R_{xx} against gate voltage V_g and magnetic field B. Numbers in the graph label the regions with corresponding filling factor. The horizontal dashed line L_2 is the conventional measurement trace of plateau transition between filling factors 5, 6, and 7, while the three tilted dashed lines are measurement paths at filling factors 6, 8, and 10, separately. Especially at the region of filling factor $\nu=6$, 6A and 6B indicate the Landau level mixing points where the R_{xx} peaks occur. Inset is the Landau level diagram at $\nu=6$.

temperature. Current amplitudes adapted to the different temperature regimes were used to avoid heating at the lowest temperatures and still ensure high enough resolution of the fluctuations at elevated temperatures.

At first we measure the phase diagram of the two-subband sample, which is the grayscale plot of longitudinal resistance R_{xx} as a function of magnetic field B and electron density n, as shown in Fig. 1. In this grayscale map, bright lines represent peaks in R_{xx}, where the electron state is delocalized, while the dark regions are the minimums, corresponding to the quantum Hall states, where localized state occurs. From this diagram we can find that the spin is resolved when the magnetic field is greater than 2.2 T. The ringlike structures at even filling factor are due to the interaction between the two sets of Landau levels.[17] This situation occurs when two Landau levels from different subbands are brought into degeneracy or mixing by tuning electron density and magnetic field. According to the standard Landau level fan diagram as illustrated schematically in the inset of Fig. 1, point 6A represents the mixing of Landau levels $(A,0,\uparrow)$ and $(S,2,\downarrow)$, while point 6B corresponds to mixing of $(A,0,\downarrow)$ and $(S,2,\uparrow)$. Here we label the single-particle levels (i,N,σ), and $i(=S,A)$, N, and $\sigma(=\uparrow,\downarrow)$ are the subband (symmetry or antisymmetry), orbital, and spin quantum numbers. Since Landau level mixing influences the pattern of phase diagram profoundly, it is interesting to find out whether this mixing will change the universality of scaling behavior of localization-delocalization transition or not.

By changing the magnetic field while varying the gate voltage, we can guide the measurement trace crossing bright lines in the phase diagram and then undergoing localized-delocalized transitions. Dashed lines in Fig. 1 illustrate such measurement traces. It should be emphasized that the tilted traces crossing ringlike structures at filling factor $\nu=6,8,10$ are important for our study since they cross the Landau level mixing points exactly. Figure 2 shows the temperature-dependence measurement of longitudinal resistance R_{xx} following trace L_1 across the Landau level mixing region at filling factor $\nu=6$. The two peaks which result from intersection of trace L_1 and the ringlike structure are noted as 6A and 6B, and each shows a broadening as the temperature increases from 120 mK to 1.2 K. In order to compare the experiment result with conventional plateau-plateau transition situation, we also measured the transition between filing factors $\nu=5,6,7$ along trace L_2. The corresponding result is plotted in Fig. 3.

Temperature scaling exponent κ can be extracted from power-law fit for the half width $\Delta B \propto T^{\kappa}$, which yields values of 0.74 and 0.77 for peaks 6A and 6B, respectively (inset of Fig. 2). It should be noticed that along line L_1 the Hall resistance remains constant since the measurement trace will not change the filling factor. On the other hand along line L_2, as illustrated in the inset of Fig. 3, the scaling exponent is extracted from half width ΔB as well as maxima of $\frac{dR_{xy}}{dB}$, which shows κ ranges within $\kappa=0.75\pm0.05$. The result is consistent with what occurs at Landau level mixing region within experiment error. The scaling in the Landau level mixing

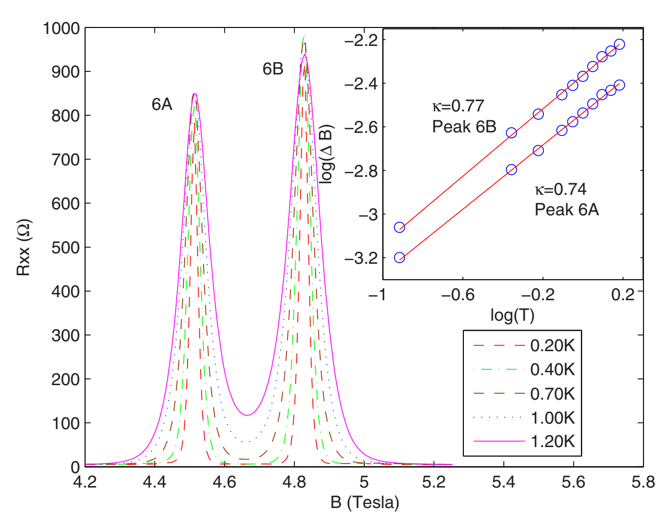

FIG. 2. (Color online) The longitudinal resistance R_{xx} at different temperatures along trace L_1. The insets show the half width ΔB vs temperature T corresponding to the two peaks in the R_{xx} curve.

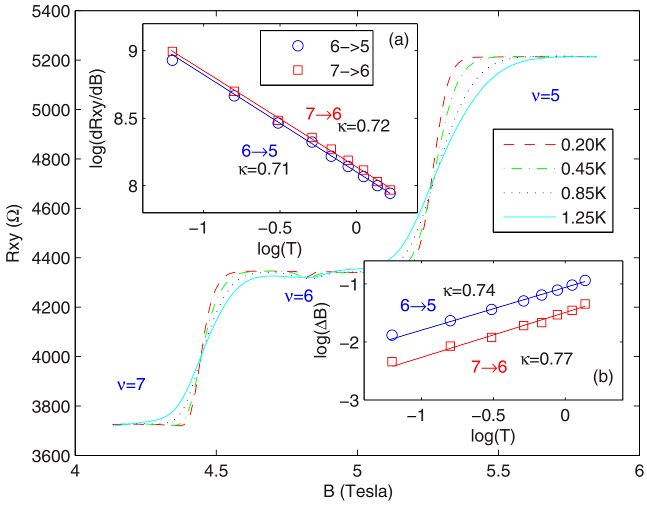

FIG. 3. (Color online) The Hall resistance R_{xy} at different temperatures along line L_2. Inset (a) shows the scaling exponent κ extracted from the temperature dependence of R_{xy} slope, while inset (b) gives κ derived from scaling of half width of R_{xx} peak.

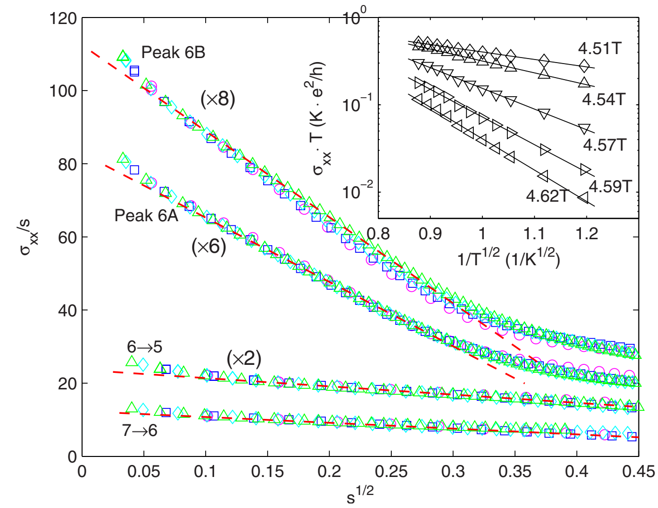

FIG. 4. (Color online) Longitudinal conductivity σ_{xx} as a function of the scaling variable s in the transition regime along line L_1 and line L_2. The dashed line is indicated for fitting to Eq. (2). The inset is the variable-range hopping fit of σ_{xx} to Eq. (1) at phase-transition region of peak 6A.

region at other filling factors ($\nu=8,10$) is also examined. All the results fall into the range $\kappa=0.75\pm0.05$. Our experiment results indicate that the exponent κ is universal with respect to the filling factor within the experimental error, whether there exists Landau level mixing or not.

Additionally, an alternative approach to scaling can be used for a direct evaluation of the localization length ξ. Polyakov and Shklovskii[18,11] argued that the mechanism for the conductivity peak broadening is variable-range hopping (VRH) in the presence of Coulomb interactions. The scaling function of VRH conductivity is given as

$$\sigma_{xx}=\sigma_0\exp\left(-\sqrt{\frac{T_0}{T}}\right),\quad T_0(B)\propto\frac{e^2}{\varepsilon\xi(B)}. \tag{1}$$

The characteristic temperature T_0 is determined by the coulomb interaction at length scale given by localization length ξ. Here $\sigma_0\propto 1/T$ is a temperature-dependent parameter. As mentioned above, the localization length $\xi(B)$ diverges as $\xi(B)\propto|B-B_c|^{\gamma}$; combining this relation yields

$$\sigma_{xx}=\sigma^{*}s\exp(-\sqrt{T^{*}s}),\quad s=\frac{|B-B_c|^{\gamma}}{T}, \tag{2}$$

where s is the scaling variable and σ^* and T^* are constants. Using this formula between conductivity σ and ξ, we can easily and directly obtain the localization length ξ and the corresponding localization exponent γ.

In order to verify that our experiment is actually in the VRH regime, the data at transition region of peak 6A are fitted according to Eq. (1) and the result is plotted in the inset of Fig. 4. The fitted straight lines demonstrate a typical VRH behavior. Therefore, the localization length exponent γ can be extracted using the scaling function (2). As shown in Fig. 4, by varying the exponent γ, all the data in the phase-transition regions along L_1, as well as L_2, can be fitted into straight lines. The perfect fit occurs only when $\gamma\approx1.3$. We notice that this result is quite different from that of previous experimental[6,9,10] and theoretical works,[19–21] which gave a value $\gamma\approx2.3$. The investigation of localization length exponent is also extended to higher filling factors in the phase diagram ($\nu=8,10$), and all the cases give a perfect fit of $\gamma=1.3\pm0.2$. The research we have done suggests that the localization length exponent γ is universal and independent of filling factor and Landau level mixing situation.

The scaling exponent κ and localization length exponent γ both show a deviation from conventional values. However, we should emphasize that the universality of the scaling behavior exists for our sample, with exponent values of $\kappa=0.75\pm0.05$ and $\gamma=1.3\pm0.2$. We can conclude that although Landau level mixing can change the degeneracy of a quantum Hall state, this mixing will not change the scaling behavior of our sample. Since the two exponents are extracted independently, we think they reveal more valuable information beneath the scaling phenomenon. Using the relation $\kappa=p/2\gamma$, we can get inelastic-scattering rate exponent $p\approx2$, which is exactly the zero-field clean limit for electron-electron scattering in two-dimensional Fermi liquids.[21–23] This result is also consistent with most of the experimental works.[8,11]

For the samples we investigated, the deviation in κ from the conventional value $\kappa\approx0.42$ can be attributed to the effect from quantum localization toward classical percolation (using the value of p and the $\kappa=p/2\gamma$ relationship, one can obtain $\kappa\approx0.42$ and $\kappa\approx0.75$ corresponding to $\gamma\approx2.4$ and $\gamma\approx1.3$, respectively). Most of the direct measurements of the localization length exponent γ yield a universal value of 2.4, which is predicted by theoretical calculations based on a network model of quantum percolation.[19–21] However, our direct measurement of γ through VRH method yields the value of 1.3, which coincides with $\gamma=4/3$, a value obtained with theories of classical percolation.[21,24] It shows that classical percolation dominates scaling behavior in our samples. Theoretical work[25] suggests that in the presence of long-range potential fluctuation due to the remote ionized impurities in AlGaAs, a larger crossover region will occur where classical percolation applies. On the other hand, in Ref. 12 the author provided a systematic investigation on the influence of the type of disorder potential on the scaling behavior. The result turned out that only for short-range alloy potential fluctuations the scaling exponent is consistent with the conventional value $\kappa \approx 0.42$. However for the long-range potential fluctuation situation, the exponent increases toward the classical value of 0.75. We think our results coincide with previous observations. This suggests that whether there exists Landau level mixing or not, the value of the scaling exponent remains the same for a given sample that contains a fixed disorder profile.

There is also a different approach for the explanation of this phenomenon. Avishai and Meir[26] studied critical exponent γ in the presence of spin-orbit scattering. Their numerical calculations show that the exponent is very close to $\gamma = 4/3$ and consistent with our results. The spin-orbit interaction is also investigated in GaAs/AlGaAs quantum well recently,[27] which suggested that this interaction arises from the structural inversion asymmetry of the heterostructure and, hence, is influenced significantly by applying gate voltage. Since our sample utilizes a front gate to tune the carrier density, it is possible for the enhancement of spin-orbit interaction, which eventually induces localization length exponent $\gamma \approx 1.3$.

In conclusion, temperature dependence of the Hall and longitudinal resistances of two-subband quantum Hall samples is measured in our experiments. The emphasis is focused on the scaling behavior of the localized-delocalized transition in the Landau level mixing regions. The scaling exponent κ is extracted by power-law fit of half width of R_{xx} peak dependence on temperature. Additionally we extract quantitative information of the localization exponent γ through an approach based on the variable-range hopping theory. The main result is that we find a universality for both the scaling exponent $\kappa \approx 0.75$ and the localization length exponent $\gamma \approx 1.3$ within the experimental error. In comparison with results obtained from conventional plateau-plateau transition, it turns out that although Landau level mixing can change the degeneracy of a quantum Hall state, the value of the scaling exponent remains the same for a given sample that contains a fixed disorder profile. Also several explanations are supposed to explain the values of γ's and κ's deviation from the conventional values.

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